**Osmotic Pressure**

If a pure solvent and a solution of a non-volatile solute in the solvent are placed on different sides of a membrane permeable only to solvent, an interesting phenomenon occurs: solvent passes through the membrane from the pure solvent side to the solution side. Why? The grass is greener on the other side of the membrane, in other words, the chemical potential of the solvent is lower in the solution than it is in the pure solvent. If passage of the solvent results in an increased pressure on the solution (due to the added column of liquid) this pressure will continue to increase until the chemical potential of the solvent in the solution is equal to that of the pure solvent. This pressure at equilibrium is called the osmotic pressure, \( \pi \).

\[
\mu^*_A(p) = \mu_A(X_A, p + \Pi) \\
\mu_A(X_A, p + \Pi) = \mu^*_A(p + \Pi) + RT \ln X_A \\
\mu^*_A(p + \Pi) = \mu^*_A(p) + \int_p^{p+\Pi} V_m dp \\
\]

solving we obtain

\[
-RT \ln X_A = \int_p^{p+\Pi} V_m dp \\
\]

replace \( \ln X_A \) with \( \ln(1 - X_B) \approx -X_B \)

obtaining

\[ RTX_B = \Pi V_m \]

For dilute solutions

\[ X_B = \frac{n_B}{n_A} \text{ and } n_A V_m = V \] so

\[ \Pi = [B]RT \] where \([B]\) is the molar concentration

or molarity of solute B. This can also be written as

\[ \Pi = \frac{c}{M}RT \] where \( c \) is the concentration of solute in grams/liter and \( M \) is the molecular weight. We can rewrite this as

\[ \frac{\Pi}{c} = \frac{RT}{M} \]

For non-ideal solutions, we can expand this equation as a virial expansion and obtain

\[ \frac{\Pi}{c} = RT(\frac{1}{M} + A_2c + A_3c^2 + ...) \]

Here \( A_2 \) is called the second virial coefficient. Often the terms above first order in \( c \) are ignored.

For solutions of polymers, \( M \) becomes \( M_n \), the number average molecular weight.

The osmotic pressure, \( \pi \), is often obtained by measuring the height of the column of solution at equilibrium. This height can be converted to pressure by multiplying by the ratio of the density of the solution to that of mercury (13.5 g/mL) and recalling one atmosphere is 760 mm Hg.