Ch 5

(Big Idea):
Small particles can go places that are impossible (classically) to reach.

Skills:
• Derive \( Y \) for 1-D particle-in-a-square-well (1-D PSW)
• Calculate tunnelling probability
1-D Particle in a Square Well

\[ E < V_0 \text{ (bound)} \]

\[ x < 0 \quad V = V_0 \]

\[ 0 < x < a \quad V = 0 \]

\[ x > a \quad V = V_0 \]

\[ 
\psi_2 \text{ is like 1-D PIB} \\
\psi_2 = A_1 e^{ikx} + A_2 e^{-ikx} \quad \left( k = \frac{\sqrt{2mE}}{\hbar} \right) 
\]

Solve for \[ \psi_2 \]

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + V_0 E = E \psi_1 
\]

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + \psi_1 (V_0 - E) = 0 \]
\[ \frac{\partial^2 \psi}{\partial x^2} + \psi(E - V_0) \frac{2m}{\hbar^2} = 0 \]

\[ S^2 + 0.5S + \frac{2m}{\hbar^2}(E - V_0) = 0 \]

\[ S^2 = \frac{2m}{\hbar^2}(\color{red}2V_0 - E) \]

\[ S = \pm \frac{\sqrt{2m(V_0 - E)}}{\hbar} \text{ note real} \]

Let us define \( j = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \)

\[ \psi_1 = Be^{jx} + Ce^{-jx} \]

by analogy

\[ \psi_3 = De^{jx} + Fe^{-jx} \]
Boundary Conditions

\(\psi_1 \rightarrow 0 \text{ as } x \rightarrow -\infty\)

So \(C = 0\)

\[
\psi_1 = Be^{jx}
\]

\(\psi_3 \rightarrow 0 \text{ as } x \rightarrow \infty\)

So \(D = 0\)

\[
\psi_3 = Fe^{-jx}
\]

5 constants: \(A, A_2, B, F, E\)

@ \(x = 0\) \(\psi_1 = \psi_2\) and \(\psi_1' = \psi_2'\)

@ \(x = L\) \(\psi_2 = \psi_3\) and \(\psi_2' = \psi_3'\)

+ normalize

\(\therefore 5 \text{ equations}\)
Non-zero probability of finding particle outside box! Despite $E < V_0$!

This is called tunneling.

Probability $\downarrow$ mass $\downarrow$ height
Tunneling

\[ \psi_3 = Fe^{-jx} \quad j = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \]

decay length \( K = \frac{\hbar}{j} \)

\[ K = \frac{\hbar}{\sqrt{2m(V_0 - E)}} \]

to tunnel:

- \( E \) must be \( \text{almost} \) \( V_0 \)
- \( m \) must be small